# Relationship between the shape of the elliptical knee joint and jumping height in a leg-type robot driven by pneumatic artificial muscle 

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#### Abstract

The McKibben pneumatic actuator (MPA) is a soft actuator used for performing various practical functions in robots. Particularly, many dynamic robots have been realized using MPAs. However, there is a trade-off between torque generated by MPA and the range of motion of the joint. In this study, we focus on the jumping motion of a leg-type robot and use an elliptical pulley whose moment arm changes depending on the robot's posture. To confirm the effectiveness of the elliptical pulley, the relationship between the knee joint pulley (patella) shape and jumping height was analyzed by simulation, and the shape of the patella maximizing jumping height was determined. It was shown that an elongated elliptical patella shape is more effective for the jumping motion than a circular one. Furthermore, the effectiveness of the analytically determined patella shape was confirmed by experiments using an actual robot.


Keywords McKibben pneumatic muscle, Leg-type robot, Jumping, Knee joint, Mechanical design

## Introduction

The McKibben pneumatic actuator (MPA) is a widely used type of actuator in robotics, which comprises a rubber tube covered with a mesh that contracts and exerts tension when filled with air (Fig. 1). The MPA is characterized by its high flexibility and weight-to-output ratio [1, 2] and has been exploited to realize dynamic motion in robots [3-12].
Owing to its flexibility, the MPA is deformed by external forces, and the output changes with deformation. This problem does not occur in robots driven by motors with high gear ratios or rigid cylinders, where deformation

[^0]does not occur. When driving a robot with MPA, the structure of the robot, such as the length and attachment position of the wires, and the shape of the joints and links, affects the deformation of the MPA. Therefore, the design of the robot structure is more significant than that of other actuators because the structure of the robot affects the output of the MPA. This study discusses the design theory of joint geometry, which significantly affects the output of the MPA and the robot's motion. It is expected that the design of joint geometry will allow the design of more diverse tension profiles of MPA.

As the MPA exerts tension only in the tensile direction, an antagonistic structure is required to drive a joint using the MPA. For example, there are methods such as attaching wires corresponding to tendons directly to links [46 ], attaching to links via pulleys [7-10], and using chains and sprockets [11, 12]. Among these, the method using pulleys has the advantages of a simple structure and easy adjustment of the moment arm. Many robots that perform dynamic movements, such as jumping and running,

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Fig. 1 Overview of McKibben pneumatic actuator: MPA (bottom) before and (top) after the application of air pressure


Fig. 2 Joint flexion (left) and extension (right) by MPA and pulley
use a structure with pulleys. As shown in Fig. 2, the joints could be extended by transmitting the tension of the MPA through the pulley.
When this structure is used, the design of the pulley is very important because of the tradeoff between the torque and range of motion. If the size of the pulley at the joint is increased to increase the torque, the magnitude of the change in the joint angle decreases with MPA contraction. The MPA has a shrinkage limit of approximately $25 \%$ for general MPA and approximately $35 \%$ for structures with superior shrinkage [13, 14]. Therefore, if the pulley's size is increased excessively, the joint's range of motion will be insufficient for movement. Hence, it is important to design a pulley that balances joint torque and range of motion.
In robots developed in previous studies [7-10], tension was transmitted through pulleys. In these studies, the pulleys were designed considering moment arms; however, the moment arms were constant because circular
pulleys were used. In contrast, a non-circular pulley has a moment arm that varies with the joint angle. This property makes it possible to achieve both torque and range of motion more suitable for robot motion than a circular pulley [15, 16]. Although the moment arm changes with joint angle even when the tendon is directly attached to the link [4-6], the structure using non-circular pulleys has an advantage in design because the motion characteristics can be changed by changing only the shape of the pulleys.

In particular, as one of the non-circular pulleys, this study focuses on elliptical pulleys. This is because the elliptical shape has fewer parameters than typical noncircular pulleys and is easy to analyze in obtaining the design principles of the pulley for the robot's motion. To confirm the effectiveness of the elliptical pulley, this study focused on the jumping motion of a leg-type robot and discussed the design theory of the pulley at the knee joint. Hereafter, the pulley at the knee joint will be referred to as the patella. Jumping motion is a dynamic motion that requires a large instantaneous force and a large range of motion of the knee joint. The balance between the torque around the knee joint and the range of motion considerably affects motion performance.
In previous studies [17, 18], jumping motion performance of leg-type robots was performed by designing the knee structure. However, the robot in [17] was driven by a hydraulic linear actuator. Although an MPA-driven robot was developed in [18], the output characteristics of the MPA, including the delay of the MPA pressure input, were not considered fully. It is expected that the jumping performance can be improved by designing a knee structure that also takes into account the output characteristics of the MPA.
In this study, we developed a leg-type robot model and mechanically analyzed the relationship between the shape of the elliptical patella and jumping height to determine the characteristics of the shape of the patella suitable for jumping. We also optimized the shape of the patella using a genetic algorithm and confirmed that the shape obtained through mechanical analysis was optimal. Finally, we conducted an experiment using an actual robot to verify the theoretical results.

## Model of jumping robot

## Structure of the robot

Here, the jumping motion of a leg-type robot was calculated via simulation. Figure 3 shows an overview of the leg-type robot model developed to analyze the patella. The robot comprised two links: an upper link corresponding to the thigh and a lower one to the crus. The waist and ankle were constrained vertically. Therefore,


Fig. 3 Structure and part names of the leg-type robot used here(left), variable names related to length (center), and variable names related to angle (right)
the robot had two degrees of freedom. This study attempted to investigate the relationship between the patellar shape and jumping height. A simple model was employed, with only the MPA for knee joint extension and a wire attached to the upper and lower links.
Each link is a uniform rigid bar, and the waist and ankle are mass points. It is assumed that masses other than the upper and lower links, waist, and ankles are negligible. Table 1 lists the length and mass values determined for the actual robot used for the experimental verification of the theoretical results. This study focused on the effect of patella shape on the jumping motion. We did change any parameters other than those related to the patella's shape.

## Shape parameters of the elliptical patella

The tension of the MPA that extends to the knee joint is transmitted through the patella. In our target robot, the
patella was fixed to the lower link, as shown in Fig. 4. The following three parameters can be used to represent an arbitrary elliptical shape. The major and minor radius of the ellipse are $a[\mathrm{~mm}]$ and $b[\mathrm{~mm}]$, respectively $(a \geq b)$, and the angle between the lower link and the major radius of the ellipse is $\alpha\left[^{\circ}\right](0 \leq \alpha<180)$ in the clockwise direction. Hereinafter, $\alpha$ is referred to as the mounting angle. When $a=b$, the shape is circular and does not depend on $\alpha$; therefore, we set $\alpha=0$. The shape of the patella is denoted as $(a, b, \alpha)$.

## Dynamical equation of the jumping robot

Figure 5 shows the forces acting on a robot. Because the friction between the wire and patella was not considered, we assumed that the tension in the MPA applied by the upper and lower links was equal to $f_{\mathrm{t}}$. Similarly, $f_{\mathrm{h}}$ is the magnitude of the tension in the wire installed

Table 1 Numerical value of each parameter of the robot

| $L_{u}$ | Upper link length | 415 mm |
| :--- | :--- | :--- |
| $L_{1}$ | Lower link length | 415 mm |
| $L_{m}$ | Length between link center axis and MPA mounting position | 23 mm |
| $L_{11}$ | Length between MPA mounting position of upper link and knee joint | 340 mm |
| $L_{12}$ | Length between MPA mounting position of lower link and knee joint | 85 mm |
| $L_{21}$ | Length between wire mounting position of upper link and knee joint | 260 mm |
| $L_{22}$ | Length between wire mounting position of lower link and knee joint | 85 mm |
| $M_{w}$ | Waist weight | 1002 g |
| $M_{u}$ | Upper link weight | 255 g |
| $M_{1}$ | Lower link weight | 297 g |
| $M_{a}$ | Ankle weight | 409 g |



Fig. 4 Definition and schematic of shape parameters $a, b, \alpha$ for elliptical patella (left) and actual robot patella used in the experiment (right)


Fig. 5 Forces acting on each part of the robot
antagonistically to the MPA for restraint purposes. The patella receives force from the wire. Let $f_{\mathrm{k}}$ be the magnitude of the force. As shown in Fig. 5, if the MPA tension received by the upper and lower links is $\overrightarrow{f_{t 1}}, \overrightarrow{\mathrm{ft}_{\mathrm{t}}}$ $\left(\left|\overrightarrow{f_{t 1}}\right|=\left|\overrightarrow{f_{t} 2}\right|=f_{\mathrm{t}}\right)$, the wire is sufficiently light, and the forces acting on the wire are balanced.

$$
\begin{equation*}
\overrightarrow{f_{\mathrm{t} 1}}+\overrightarrow{f_{\mathrm{t} 2}}+\overrightarrow{f_{\mathrm{k}}}=\overrightarrow{0} \tag{1}
\end{equation*}
$$

is satisfied. The directions of $\overrightarrow{f_{\mathrm{t} 1}}$ and $\overrightarrow{f_{\mathrm{t} 2}}$ are the directions of the tangent lines drawn from the MPA attachment points of the upper and lower links to the elliptical patella. Therefore, the tension values can be obtained geometrically based on the robot's posture and patella's shape.
The robot was initially supported by the ankle on the floor and the waist on a stopper fixed at a height of 0.5 m from the floor. This stopper was intended to unify the initial height and initial velocity during the experiment and
improve the reproducibility; it only supported the waist from below before the jumping and did not affect the motion of the robot once the waist begins to move. From the initial state, until the waist leaves the stopper, it is subjected to a reaction force from the stopper. The magnitude of this force was $N_{\mathrm{w}}$. The ankle part is subjected to a reaction force from the floor until takeoff. Let $N_{\mathrm{a}}$ be the magnitude of the force. Let $\left(F_{\mathrm{wx}}, F_{\mathrm{wy}}\right)$ be the forces between the waist and the upper link, ( $F_{\mathrm{kx}}, F_{\mathrm{ky}}$ ) be the forces between the upper and lower links, and ( $F_{\mathrm{ax}}, F_{\mathrm{ay}}$ ) be the forces between the lower link and the ankle. The angle between the tension of the MPA and upper link is defined as $\gamma_{11}$, and the angle between the MPA and lower link is $\gamma_{12}$, as shown in Fig. 3. Similarly, the angle between the wire tension and upper link is defined as $\gamma_{21}$, and the angle between the wire tension and lower link is defined as $\gamma_{22}$. The angles of the upper and lower links of the robot made in the horizontal plane are defined as $\eta_{1}$ and $\eta_{2}$, respectively.
The equations of motion were derived from the forces acting on each part. In Fig. 5, $x$-axis is directed in the right direction and $y$-axis is directed upward. The rotation was assumed to be counterclockwise in the positive direction. The waist moves only in the direction of the $y$-axis because of the vertical constraint. The equation of motion is as follows:

$$
\begin{equation*}
M_{\mathrm{w}} \ddot{y}_{\mathrm{w}}=F_{\mathrm{wy}}+N_{\mathrm{w}}-M_{\mathrm{w}} g \tag{2}
\end{equation*}
$$

From Fig. 5, the equation of motion of the upper link in the $x$-direction is
$M_{\mathrm{u}} \ddot{x}_{\mathrm{u}}=-F_{\mathrm{wx}}+F_{\mathrm{kx}}+f_{\mathrm{t}} \cos \left(\eta_{1}-\gamma_{11}\right)+f_{\mathrm{h}} \cos \left(\eta_{1}+\gamma_{21}\right)$
The equation of motion in the $y$-direction is
$M_{\mathrm{u}} \ddot{y}_{\mathrm{u}}=-F_{\mathrm{wy}}+F_{\mathrm{ky}}-f_{\mathrm{t}} \sin \left(\eta_{1}-\gamma_{11}\right)-f_{\mathrm{h}} \sin \left(\eta_{1}+\gamma_{21}\right)-M_{\mathrm{u}} g$

The equation of rotation around the center of gravity is

$$
\begin{align*}
-I_{\mathrm{u}} \ddot{\eta}_{1}= & \frac{1}{2} F_{\mathrm{wx}} L_{\mathrm{u}} \sin \eta_{1}+\frac{1}{2} F_{\mathrm{wy}} L_{\mathrm{u}} \cos \eta_{1} \\
& +\frac{1}{2} F_{\mathrm{kx}} L_{\mathrm{u}} \sin \eta_{1}+\frac{1}{2} F_{\mathrm{ky}} L_{\mathrm{u}} \cos \eta_{1} \\
& -f_{\mathrm{t}}\left\{\left(L_{11}-\frac{1}{2} L_{\mathrm{u}}\right) \sin \gamma_{11}+L_{\mathrm{m}} \cos \gamma_{11}\right\} \\
& +f_{\mathrm{h}}\left\{\left(L_{21}-\frac{1}{2} L_{\mathrm{u}}\right) \sin \gamma_{21}+L_{\mathrm{m}} \cos \gamma_{21}\right\} \tag{5}
\end{align*}
$$

From Fig. 5 and Equation (1), the equation of motion of the lower link in the $x$-direction is
$M_{1} \ddot{x}_{1}=-F_{\mathrm{kx}}+F_{\mathrm{ax}}-f_{\mathrm{t}} \cos \left(\eta_{1}-\gamma_{11}\right)+f_{\mathrm{h}} \cos \left(\eta_{2}+\gamma_{22}\right)$
The equation of motion in the $y$-direction is
$M_{1} \ddot{y}_{\mathrm{l}}=-F_{\mathrm{ky}}+F_{\mathrm{ay}}+f_{\mathrm{t}} \sin \left(\eta_{1}-\gamma_{11}\right)+f_{\mathrm{h}} \sin \left(\eta_{2}+\gamma_{22}\right)-M_{1} g$

The equation of motion for rotation around the center of gravity is

$$
\begin{align*}
I_{1} \ddot{\eta}_{2}= & \frac{1}{2} F_{\mathrm{kx}} L_{1} \sin \eta_{2}-\frac{1}{2} F_{\mathrm{ky}} L_{1} \cos \eta_{2} \\
& +\frac{1}{2} F_{\mathrm{ax}} L_{1} \sin \eta_{2}-\frac{1}{2} F_{\mathrm{ay}} L_{1} \cos \eta_{2} \\
& +f_{\mathrm{t}}\left\{L_{11} \sin \gamma_{11}+L_{\mathrm{m}} \cos \gamma_{11}+\frac{1}{2} L_{1} \sin \left(\eta_{1}+\eta_{2}-\gamma_{11}\right)\right\} \\
& +f_{\mathrm{h}}\left\{\left(\frac{1}{2} L_{1}-L_{22}\right) \sin \gamma_{22}-L_{\mathrm{m}} \cos \gamma_{22}\right\} \tag{8}
\end{align*}
$$

Finally, from Fig. 5, the equation of motion of the ankle in the $y$-direction is

$$
\begin{equation*}
M_{\mathrm{a}} \ddot{y}_{\mathrm{a}}=-F_{\mathrm{ay}}+N_{\mathrm{a}}-M_{\mathrm{a}} g \tag{9}
\end{equation*}
$$

where $I_{\mathrm{u}}$ and $I_{1}$ are the moments of inertia around the center of gravity of the upper and lower links, respectively $I_{\mathrm{u}}=\frac{1}{12} M_{\mathrm{u}} L_{\mathrm{u}}{ }^{2}$ and $I_{\mathrm{l}}=\frac{1}{12} M_{\mathrm{l}} L_{\mathrm{l}}{ }^{2}$ for a uniform rigid bar. The forces acting on each part of the robot can be obtained using Equations (2-9):

## Tension model of MPA

Table 2 lists the dimensions and materials of the MPAs used here. For information on the output characteristics of the MPAs, refer to [19].
Various tension models have been developed for MPAs [20-23]. This study adopted the "Linear approximation model" proposed in a previous study [24]. This model accurately represents the tension of MPAs based on pressure and length. In the linear approximation model, the tension $f_{\mathrm{t}}$ of the MPA is

Table 2 Properties of the MPA used in this study

| Length of MPA (when pressure is not input) | 250 mm |
| :--- | :--- |
| Weight of MPA (when pressure is not input) | 41.4 g |
| Inner diameter of rubber tube | 9 mm |
| Thickness of rubber tube | 1.5 mm |
| Material of rubber tube | silicone rubber |
| Material of mesh sleeve | polyester |

$$
\begin{equation*}
f_{\mathrm{t}}=S_{1} P+S_{2} P L+S_{3} L+S_{4}+\gamma V \tag{10}
\end{equation*}
$$

In Equation (10), $P$ is the pressure of the MPA, $L$ is the length of the MPA, and $V$ is the contraction velocity of the MPA. $S_{1}, S_{2}, S_{3}, S_{4}$ and $\gamma$ are constants determined from the initial length, diameter, and material of the MPA. Here, $S_{1}=-0.0036 \mathrm{~m}^{2}, S_{2}=0.0207 \mathrm{~m}$, $S_{3}=0.0078 \mathrm{~N} / \mathrm{m}, S_{4}=-0.0020 \mathrm{~N}$, and $\gamma=-35.0 \mathrm{Ns} / \mathrm{m}$.

## Pressure input

A strong instantaneous force is required to cause the robot to jump. Therefore, a step-like pressure input is typically applied. However, in reality, there is a delay in the input owing to the flow path length and other factors. Therefore, the time variation in the pressure $P(t)$ in the simulation is defined as follows:

$$
\begin{equation*}
P(t)=P_{\max }\left(1-e^{-\frac{t}{\tau}}\right) \tag{11}
\end{equation*}
$$

Here, $P_{\max }=0.4 \mathrm{MPa}$ and $\tau=0.085 \mathrm{~s}$. This study attempts to investigate changes in jumping motion based on the patella's shape. Therefore, the values of the pressure inputs shown in Equation (11) were used in all the simulations.

## Tension model of antagonist wire

The tension from the MPA alone cannot be used to realize a jumping motion because the upper and lower links do not move in tandem, causing the knee joint to extend by more than $180^{\circ}$. Therefore, a wire was attached in antagonism to the MPA for restraint (the Wire in Fig. 3). The tension $f_{\mathrm{h}}$ of the wire is

$$
f_{\mathrm{h}}= \begin{cases}0 & \left(d_{\mathrm{w}} \leq l_{\mathrm{w}}\right)  \tag{12}\\ K_{\mathrm{w}}\left(d_{\mathrm{w}}-l_{\mathrm{w}}\right)+C_{\mathrm{w}} \dot{d_{\mathrm{w}}} & \left(d_{\mathrm{w}}>l_{\mathrm{w}}\right)\end{cases}
$$

where $d_{\mathrm{w}}$ is the length between the fixed positions of the wire and $l_{\mathrm{w}}$ is the natural length of the wire. If $d_{\mathrm{w}} \leq l_{\mathrm{w}}$, the wire is slack, and the tension is zero. If $d_{\mathrm{w}}>l_{\mathrm{w}}$, then the wire is stretched beyond its natural length and exerts tension proportional to its length and speed. Here, $l_{\mathrm{w}}=0.28 \mathrm{~m}, K_{\mathrm{w}}=5.0 \times 10^{3} \mathrm{~N} / \mathrm{m}$, and $C_{\mathrm{w}}=5.0 \times 10^{2} \mathrm{Ns} / \mathrm{m}$.

## Relationship between patella shape and robot jumping height

We tested different patellar shapes to maximize the jump height of the leg-type robot. First, we discuss two mechanical requirements that are considered important for a leg-type robot to perform high jumps. The word "requirement" in this paper is not used in the sense of a mathematically rigorous condition or threshold but rather in the sense of a favorable tendency to achieve high jumps. Next, the shape of the patella satisfying these requirements is discussed. Subsequently, we examined whether the shape of the patella obtained via analysis was suitable for jumping motion using simulations.

## Requirement 1 : increase in the internal pressure of MPA in the initial position

A leg-type robot was initially supported by a stopper at the waist and began to move when the tension in the MPA caused $F_{\text {wy }}$ in Equation (2) to exceed $M_{\mathrm{w}} g$ and the reaction force $N_{\mathrm{w}}$ from the stopper became zero. In the jumping motion considered here, the tension of the MPA is expressed by Equation (10), and the pressure input to the MPA is expressed by Equation (11). According to Equation (10), the tension in the MPA increases with the pressure and MPA. Moreover, from Equation (11), the internal pressure of the MPA gradually increases after the start of the pressure input. Therefore, the later $F_{\mathrm{wy}}$ exceeds $M_{\mathrm{w}} g$, the greater the increase in the internal pressure of the MPA during that time. Thus, the MPA can exert higher tension from the start of the movement to takeoff, which is considered effective for improving the jumping height. To achieve this, the patella's shape should be designed such that the force of lifting the waist, $F_{\mathrm{wy}}$, is small relative to the tension of the MPA, $f_{\mathrm{t}}$, in the initial state.

## Requirement 2: variation in waist acceleration with time

The robot's knee joint was extended by the tension of the MPA, and the antagonistically placed wires for restraint exerted the tension shown in Equation (12). This causes the floor reaction force $N_{\mathrm{a}}$ to be zero, resulting in the takeoff of the robot. The wires begin to exert tension, independent of the patella's shape, when the waist height reaches a certain value. Because the restraining wire exerts a large tension instantaneously, takeoff occurs immediately after wire tensioning. The robot is affected only by gravity after takeoff; therefore, the robot needs to have a high velocity in the vertical center of gravity at takeoff for a high jump. As the ankles are grounded until takeoff, the center-of-gravity velocity can be increased by increasing the waist velocity.
Waist velocity at takeoff depends on the waist acceleration between the initial posture and takeoff. Let $y_{\mathrm{w}}(t)$
be the height of the waist at time $t$ and $h$ be the height of the waist at the moment when the restraining wire is stretched. The robot exits immediately after $y_{\mathrm{w}}(t)=h$. Let $t=t_{1}$ be the time point at that instant. $\dot{y}_{\mathrm{w}}\left(t_{1}\right)$ should be sufficiently large to perform a high jump. Here, the waist acceleration $\ddot{y}_{\mathrm{w}}(t)$ must increase the value of $\dot{y}_{\mathrm{w}}\left(t_{1}\right)$ under the condition that $y_{\mathrm{w}}\left(t_{1}\right)=h$. In other words, the value of $\ddot{y}_{\mathrm{w}}(t)$ should be small at small $t\left(0 \leq t \leq t_{1}\right)$, and $\ddot{y}_{\mathrm{w}}(t)$ should be large at large $t$. The waist height $y_{\mathrm{w}}(t)$ is a double integral of the acceleration $\ddot{y}_{\mathrm{w}}(t)$. If the value of $\ddot{y}_{\mathrm{w}}(t)$ is large at small $t$, the waist velocity $\dot{y}_{\mathrm{w}}(t)$ increases at an early stage, and the value of $y_{\mathrm{w}}(t)$ soon reaches $h$. Simultaneously, if the value of $\ddot{y}_{\mathrm{w}}(t)$ is small at a small $t$, the increase in $y_{\mathrm{W}}(t)$ is suppressed, and $\dot{y}_{\mathrm{w}}(t)$ increases.

Examples of $y_{\mathrm{w}}(t), \dot{y}_{\mathrm{w}}(t)$, and $\ddot{y}_{\mathrm{w}}(t)$ are shown in Fig. 6. Figure 6 shows two types of acceleration and an example of the time variation in velocity and waist height. Condition 1 is the case where $\ddot{y}_{\mathrm{w}}(t)$ tends to be large at large $t$, and Condition 2 is the case where $\ddot{y}_{\mathrm{w}}(t)$ tends to be large at small $t$. A comparison of the velocities when the waist reaches a certain height ( 0.69 m ) shows that under Condition 1, the velocity is higher than under Condition 2. Thus, by taking a large $\ddot{y}_{\mathrm{w}}(t)$ at large $t$, the velocity of the waist can be efficiently increased until $y_{\mathrm{w}}(t)=h$. The robot jumps by gradually extending its knee joint from the flexed position. Therefore, $\ddot{y}_{\mathrm{w}}(t)$ should be small when the robot's knee joint is in a flexed position, and $\ddot{y}_{\mathrm{w}}(t)$ should increase rapidly with extension. As the motion of the waist is expressed by Equation (2), to realize such a tendency in $\ddot{y}_{\mathrm{w}}(t)$, the patella's shape should be designed such that the waist lifting force $F_{\text {wy }}$ relative to the MPA tension $f_{\mathrm{t}}$ increases with the extension of the knee joint.

## Design policy of elliptical patella for high jump

In the leg-type robot, $F_{\mathrm{wy}}$ increased with $\gamma_{11}$. Based on the discussion in the previous subsection, $\gamma_{11}$ should be small in the initial state to satisfy Requirement 1, and $\gamma_{11}$ should increase with knee joint extension to satisfy Requirement 2.
The shape of the patella satisfying these two requirements is also discussed. First, if the patella is circular $(a=b), \gamma_{11}$ does not change depending on the robot's posture. Considering Requirement 2, $\gamma_{11}$ should change significantly in response to the robot's posture. Therefore, it is important that the difference between the major radius $a$ and minor radius $b$ be sufficiently large. Here, we compare two conditions: one in which the difference between $a$ and $b$ is large, $(a, b)=(80,10)$ and the other in which the difference is small, $(a, b)=(80,60)$. Figure 7 shows the relationship between the robot waist height $y_{\mathrm{w}}$ and $\gamma_{11}$ when the mounting angle $\alpha$ varies in $15^{\circ}$ increments for cases $(a, b)=(80,10)$ and $(80,60)$. In the case


Fig. 6 Time variation in velocity and height with respect to time variation in acceleration under two waist conditions. Velocity at takeoff ( $h=0.7 \mathrm{~m}$ ) is greater in Condition 1 than in Condition 2. Condition 1(Blue line): $\ddot{y} w(t)$ is large at large $t$ Condition 2(Red line): $\ddot{y} w(t)$ is large at small $t$
of $(80,10)$, the variation in $\gamma_{11}$ with respect to $y_{\mathrm{w}}$ is larger than that in the case of $(80,60)$. Considering Requirement 1 , a smaller value of $\gamma_{11}$ at the initial posture $y_{\mathrm{w}}=0.5 \mathrm{~m}$ is better; therefore, a range of $\alpha$ from 60 to $90^{\circ}$ is preferred. Considering Requirement 2 , the range of $\alpha$ should be $15-75^{\circ}$ because $\gamma_{11}$ should have an increasing trend with
respect to $y_{\mathrm{w}}$. Based on the above, the shape of the patella suitable for jumping exercises has an elliptical shape with the following characteristics: first, the difference between $a$ and $b$ should be large; second, $\alpha$ should be in the range of approximately $60-75^{\circ}$.


Fig. 7 Comparison of the relationship between the waist height $y_{w}$ and $\gamma_{11}$ for different $a, b$ and $\alpha$

Validation of the analysis of patella shape via simulation The validity of the analytically derived patellar geometry for the jumping motion was verified via simulation. Table 3 lists the shapes and mechanical and geometric characteristics of the seven considered patellar shapes. This verification aimed to investigate the relationship between patellar shape and jumping performance. Therefore, the pressure input to the MPA and the other initial conditions did not change. The length of the wire transmitting the MPA tension was set such that there would be no stretch or slack in the initial posture of each patella supported at the waist.
Figure 8 shows the highest point reached by the center of gravity in the jumping motion for seven different patellar shapes. The analytically derived patellar shape
$(100,20,70)$ results in the highest jump. The jumping motion is illustrated in Fig. 9. Figure 10 shows the time variation in waist height. The waist starts to move later with patellas of $(100,20,70),(50,20,70)$, and $(20,20,0)$ satisfy Requirement 1 . However, the subsequent increase in waist height differs with patella ( $100,20,70$ ), which also satisfies Requirement 2, resulting in the highest jump. In contrast, when only Requirement 2 is satisfied, $(100,20,30)$, the waist starts to move earlier, and the tension of the MPA is not fully utilized. These results confirm that Requirements 1 and 2 are important and that the patellar geometry obtained in the analysis is effective for high jumps.

Table 3 Seven compared patella shapes and their mechanical and geometric characteristics

| No. | $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{\alpha})$ | Geometrical and mechanical characteristics |
| :--- | :--- | :--- |
| 1 | $(100,20,30)$ | Elongated elliptical shape satisfying only Requirement 2 |
| 2 | $(100,20,70)$ | Ideal elliptical shape satisfying both Requirements 1 and 2 |
| 3 | $(100,20,165)$ | Unsuitable for jumping that does not satisfy both Requirements 1 |
| and 2 Elliptical shape |  |  |
| 4 | $(50,20,70)$ | Similar to an ideal elliptical shape, but $a$ is not extremely large |
| 5 | $(100,50,70)$ | Similar to an ideal elliptical shape, but b is not extremely small |
| 6 | $(20,20,0)$ | Circular patella with a small radius that satisfies only Requirement 1 |
| 7 | $(100,100,0)$ | True circular shape with a large radius |



Fig. 8 Highest point reached by the center of gravity in jumping simulations with seven different patellas


Fig. 9 The simulation of jumping when using $(a, b, \alpha)=(100,20,70)$ patella


Fig. 10 Temporal variation in waist height in the jumping simulation for seven different patellas

## Optimization of patella shape using genetic algorithm

In this section, we describe the use of a genetic algorithm to optimize the shape of the patella to maximize the jump height. The validity of the analysis was confirmed by examining whether the optimized shape satisfied the conditions mentioned in the previous section. The objective function of the optimization is the height reached by the robot's center of gravity during a jump. The variables to be optimized are the three parameters $(a, b, \alpha)$, which describe the shape of the elliptical patella. Owing to the constraints of the actual robot design, $(a, b, \alpha)$ is set to $20 \leq a \leq 100,20 \leq b \leq 100$, and $0 \leq \alpha<180$. Optimization was performed using 50 individuals and 50 generations and converged at $(a, b, \alpha)=(98,20,70)$. The optimized patella is an elongated ellipse with a large difference between $a$ and $b$, and the value of $\alpha$ is within the range $60-75^{\circ}$. This result confirms that the shape
obtained analytically is the most suitable for high jumps among arbitrary elliptical shapes.

## Jumping experiment using an actual leg-type robot

Leg robot and experimental methods
This section describes the experimental verification of the effectiveness of the patellar shape obtained analytically for actual jumping. Seven types of patellas listed in Table 3 were used in the experiment. The actual patellas were manufactured using a 3D printer, as shown in Fig. 11. Figure 12 presents an overview of the leg-type robot used in this experiment. The upper and lower links were made of carbon pipes because of their high strength and low weight. Like the robot model shown in Fig. 3, the waist and ankle parts were vertically constrained by a slider mechanism. The lengths of the robot links match the values in Table 1.

The following procedure was used to compare the jumping heights attained using seven patellas. The waist of the robot was supported by a stopper positioned 0.5 m from the floor, and the robot was initially in a completely stationary state. From this state, a step-like pressure input up to 0.4 MPa was applied to the MPA to initiate a jump. Waist and ankle positions were measured using a motion capture system. Jumping experiments were performed 15 times for each patella.

## Experiment results

Figure 13 shows the highest point of the robot's center of gravity for each patella ( 15 jumps). The highest jump was observed when the ideal shape $(100,20,70)$ was used, confirming that the elliptical patella obtained via


Fig. 11 Seven prototype patellas for the experiment. These were manufactured from PLA using a 3D printer. Patella numbers in the figure correspond to those in Table 3
the analysis was effective for actual jumping. Figure 14 shows a jumping motion of the robot. Figure 15 shows the temporal variation in waist height for each patella. Similar to the simulation results, because the force lifting the waist is small in the initial posture, the waist begins to move later when using the ( $100,20,70$ ), $(50,20,70)$, and $(20,20,0)$ patellas that satisfy Requirement 1 . Among these three patellar shapes, the degree to which they satisfy Requirement 2 is correlated with the jump height. However, in actual equipment, $\gamma_{11}$ is not as small as in
the simulation because of the size of the MPA itself. The function of increasing the internal pressure of the MPA as indicated in Requirement 1, is considered to be smaller than in the simulation. Therefore, the jumping heights of these three patellas would tend to be lower than those in the simulation. The difference between simulation and experiment may be partly due to modeling errors in the MPA. For example, in the "linear approximation model" used in this simulation, the tension value varies depending on the value of the length parameter. A more detailed discussion of these differences is an issue to be addressed in the future. At the same time, $(100,20,30)$ satisfies Requirement 2 but does not satisfy Requirement 1, as in the simulation. Therefore, the internal pressure of the MPA could not be increased before the start of the movement, resulting in a sub-maximal jump height. These results confirm that the elliptical patella shape is effective for the jumping motion of an actual leg-type robot and that Requirements 1 and 2 are both important for maximizing the jump height.

## Conclusion

This study attempted to investigate the effectiveness of elliptical pulleys in MPA-driven robots. Specifically, the shape of the patella, which affects the range of motion and extension torque of the joint, was optimized to maximize the jump height of the leg-type robot. Mechanical analysis showed that the following two points are important for achieving a high jump in a leg-type robot. First, the force lifting the waist should be small in the


Fig. 12 Overview of the developed leg-type robot and its components


Fig. 13 Box-and-whisker diagram showing the highest point reached by the center of gravity ( 15 jumps) for each patella in the actual experiment


Fig. 15 Variation in waist height with time for each patella. (data for one representative jump out of 15 jumps)


Fig. 14 Jumping motion of a leg-type robot with patella parameters of $(a, b, \alpha)=(100,20,70)$
bent-knee position. Second, the force lifting the waist should increase with knee extension because greater acceleration in the second half of the knee joint extension results in a higher waist velocity before takeoff. An elliptical patella with a large major radius and small minor radius attached at a fixed angle $\left(60-75^{\circ}\right.$ in the case of our robot) to the lower link was considered to satisfy these requirements. The effectiveness of the analytically derived patellar parameters for jumping motion was confirmed using a genetic algorithm. Furthermore, the proposed shape exhibited high jumping performance in an experiment using an actual robot. In future, we will investigate whether the same approach can be used to improve the performance of other types of locomotion in robots.

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## Author contributions

TO conducted the simulation analysis and experiments. TO, DN, KN and YS wrote the manuscript. All authors have discussed the results and approved the final manuscript. All authors read and approved the final manuscript.

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## Availability of data and materials

The data supporting the findings of this study are available from the corresponding author, YS, upon reasonable request.

## Decarations

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